The Dynamics of Small Bodies

Dissipative and Radiation Forces

Props: flashlight, milk, water, glass containers, polarizers, red and green lasers
Additional Forces on Small Bodies

For planets, gravity is the largest force. However small bodies can be significantly affected by:

1. Radiation pressure from the Sun. Pushes the particle outward.
2. Poynting-Robertson drag, a relativistic radiative effect which causes cm-sized (and smaller) particles to spiral inward toward the Sun.
3. The Yarkovski effect. Changes the orbits of m to km sized objects due to uneven temperature distributions at their surface.
4. Gas drag. For particles smaller than one µm, the solar wind can cause corpuscular drag. Gas drag causes orbits of satellites to decay if they reach the upper atmosphere. Could have been very important for protosolar nebula.
5. Electromagnetic forces. Important for small charged particles.
Radiation Pressure

• Photons from the Sun carry momentum.

\begin{align*}
\text{photon wavelength } \lambda \\
\text{frequency } \nu \\
\text{c speed of light}
\end{align*}

\begin{align*}
E &= h\nu \\
h &= \text{Planck’s constant} \\
p &= \frac{h\nu}{c}
\end{align*}

• When a particle absorbs momentum it feels a force. Pressure is force per unit area. Absorption of radiation from the Sun results in \textbf{radiation pressure}. 
Flux from the Sun

**Luminosity** of the Sun = energy emitted per second integrated over all wavelengths

\[ L_\odot = 3.8 \times 10^{33} \text{ erg/s} \]

**Flux** = energy per second going through a particular area (integrated over all wavelengths)

Flux received at Earth from the Sun

\[ F = \frac{L_\odot}{4\pi r^2} \]

\( r \) is distance of Earth from Sun

units: \( \text{erg s}^{-1} \text{ cm}^{-2} \)
Force due to Radiation Pressure

The momentum of a photon is energy divided by $c$ the speed of light \( p = E/c \)
Force is \( dp/dt \) So radiation force is momentum of absorbed photons per unit time

Total radiation force \( F_{\text{rad}} = \frac{f}{c} A Q \)
energy in photons per unit time and area

\( f \) = energy flux
\( f = \frac{L_\odot}{4\pi r^2} \)
\( r = \text{distance to Sun} \)

\( A = \pi a^2 \) area of particle absorbing radiation, particle radius \( a \)

\( F_{\text{rad}} = \frac{L_\odot}{4\pi r^2 c} \pi a^2 Q \)
The ratio of radiation and gravity forces

The force due to gravity also depends on $r^{-2}$
Ratio of force due to radiation pressure compared to that due to gravity is unitless parameter $\beta$

\[
F_{\text{rad}} = \frac{L_{\odot}}{4\pi r^2 c} \pi a^2 Q
\]
\[
F_{\text{grav}} = \frac{GM_{\odot}m}{r^2}
\]
\[
m = \frac{4\pi}{3} \rho a^3 \quad \text{mass of particle}
\]
\[
\beta \equiv \frac{F_{\text{rad}}}{F_{\text{grav}}} = \frac{3L_{\odot}Q}{16GM_{\odot}c\rho\pi a}
\]

independent of $r$
Particle under the forces of gravity and radiation pressure only

The force from radiation pressure has the same dependence on radius as Gravity, but is in the opposite direction.

\[ F = -\frac{GMm}{r^2} \hat{r} \]  for gravity alone

\[ = (\beta - 1)\frac{GMm}{r^2} \hat{r} \]  for gravity and radiation pressure

If \( \beta > 1 \) the force is repulsive and the particle is expected from the system
Radiation pressure — is it important? depends on particle radius

By inserting fundamental constants and astronomical quantities such as the luminosity and mass of the Sun, we find:

$$\beta = 1\left(\frac{a}{0.1\mu m}\right)^{-1}\left(\frac{Q}{0.5}\right)\left(\frac{\rho}{3g\ cm^{-3}}\right)^{-1}$$

Particles smaller than 0.1µm will be expelled immediately. Particles larger than this will orbit the sun, but they will do so as if they were in a system with reduced Gravity.

Note $\beta$ also depends on the mass and luminosity of the star. Here I have assumed solar mass and luminosity.
Poynting-Robertson (PR) Drag

- A particle in a heliocentric orbit that reradiates solar energy isotropically in its own frame of reference, actually emits more momentum in the forward direction as seen in the solar reference frame.
- This is because the frequencies of photons emitted in this direction are relativistically boosted or blueshifted.
Now add the slight complication of revolution. If the body’s orbital velocity is $v$, then from the body’s point of view the star moves at $-v$, and thus the light has a component of velocity anti-parallel to the body’s velocity.
PR drag (continued)

\[ \mathbf{F}_{Rad} = \beta \frac{G M m}{r^2} \left[ (1 - \frac{2v_r}{c})\mathbf{\hat{r}} - \frac{v_\theta}{c} \mathbf{\hat{\theta}} \right] \]

where the first term is the radiation pressure, the second two terms are the PR drag.

- The direction of the force depends on the particle’s direction of motion. For a circular orbit it is opposite to the particle’s direction of motion. The particle is slowly slowed down.
- PR drag is a relativistic effect and so is proportional to \(v/c\). Because the force depends on velocity, it is a dissipative force, like friction or drag. Energy is removed from the particle.
- Even though \(v/c\) is small, over a long period of time PR drag causes small particles to spiral into the Sun (where it is sublimated and blown away).
PR Drag Decay Timescale

\[ F_{PR} \sim -\beta \frac{v}{c} F_{\text{gravity}} \] to order of magnitude

\[ t_{PR} \sim t_{\text{gravity}} \left( \beta \frac{v}{c} \right)^{-1} \] timescale for PR drag to be important

\[ t_{\text{gravity}} \sim P \] Rotation period

Scaling from velocity of Earth in orbit (30km/s)

\[ \frac{v}{c} = \frac{3 \times 10^6 \text{cm/s}}{3 \times 10^{10} \text{cm/s}} = 10^{-4} \]

Orbital period 1 year

\[ t_{PR} \sim 10^4 \text{years} \left( \frac{r}{1 \text{AU}} \right)^2 \beta^{-1} \]

\[ v_c = \sqrt{\frac{GM_*}{r}} \propto r^{-1/2} \]

\[ P = \frac{2\pi r^{3/2}}{(GM)^{1/2}} \]
PR Decay Timescale

- PR decay is very short compared to the age of the solar system.
- Dust particles must be continually produced.
- Dust particles are produced by collisions in the asteroid and Kuiper Belt, and by outgassing of comets.
The Zodiacal Cloud

Dust in the asteroid belt is known as the Zodiacal Cloud. It reflects sunlight and can be seen at a very dark site the hour after sunset and before sunrise.

The zodiacal cloud is hot enough to emit light in the infrared and was seen in all sky infrared maps from IRAS and COBE satellites.

Photo credit Chris Proctor
Zodiacal light

Taken in Namibia, with a fisheye lens, by Stefan Seip.
The Earth’s Resonant Dust Ring

The slow spiral in of dust particles can be delayed if the particle is trapped in a resonance with a planet. Kicks from the resonance oppose the drag force. Because particles can spend along time trapped in resonances, there can be a lot of dust particles at these locations.

Figure Credit Marc Kuchner
The trailing side is predicted to be brighter than the leading side. This has been observed in IRAS and COBE far infrared images (zodiacal model subtracted).
Dust in the Kuiper Belt

Dust, is predicted to be in the Kuiper belt. From an outside perspective, the spatial distribution could reveal the presence of the outer planets.

Credit Liou and Zook
Debris Disks, Vega-type stars

IRAS discovered that some stars were thousands of times brighter than expected in the mid-far infrared.

Dust absorbing the light from the star reradiates it at lower temperatures, in the infrared. The amount of dust is 10,000 times more than in our Zodiacal cloud. Because the lifetime due to PR drag is short, this dust must be replenished by collisions. They are dubbed “debris” disks.
Debris disks can have spectra similar to dusty comets.
Beta Pic, a case study

• Beta Pic’s debris disk also has a hole in it, as inferred from the lack of hotter dust.
• The disk is warped.
• Both phenomena could be explained by the presence of planets.
• A massive planet has recently been seen in imaging studies inside the hole!
• Beta Pic exhibits transient absorption lines, which could be caused by evaporating star grazers (FEBs, falling evaporating bodies).
• Beta Pic has 1 Earth mass of dust, with a similar spectrum to cometary dust (silicates), corresponding to a suspected planetesimal mass that is 10000 more times massive as that remaining in our solar system.
• Beta Pic is only $2 \times 10^8$ years old, much younger than our solar system.
Determining Asteroid Sizes

Visible Light
- High Albedo "Chalk"
- Low Albedo "Charcoal"

Brightness alone does not correspond to size

Infrared Light
- High Albedo "Chalk"
- Low Albedo "Charcoal"

Brightness corresponds to size
Yarkovski Effect

- Warm areas on asteroids emit more photons than cooler areas. These areas are warmest on the evening side of the asteroid.
- Because an asteroid's surface gets hotter the longer sunlight falls on it, it does not reradiate energy evenly throughout its day or year.
- The asteroid will receive a net kick in a particular direction, just as a rocket spewing a jet of gas recoils in the opposite direction.
The thrust depends on whether the asteroid is prograde or retrograde and on the obliquity.

For prograde rotation the force is positive, for retrograde motion it is negative and similar to PR drag.
Yarkovsky effect

from Bottke et al. 2006
Yarkovsky effect

• Diurnal -- rotating asteroid
  – dusk side is hotter, so emits more radiation
  – Relativistic effect causing changes in semi-major axis.
  – Retrograde rotators spiral inwards

• Seasonal
  – dusk side again hotter, always leading to in-spiraling.
Yarkovsky effect

Skin depth (how deep is hot layer after heating by 1 day)
Cooling timescale (energy absorbed is radiated away at night, how long does this take?)

If spin is fast compared to cooling time then whole body is the same temperature (and no drift!)

Radiation pressure (and orbital drift rate) depends on temperature variation on surface
Drift Rates of NEOs from main belt

The spin period $P_{\text{rot}}$ is 6h for bodies larger than 0.15 km in diameter and $P_{\text{rot}} = 6h \times (D/0.15 \ km)$ for smaller bodies.

O’Brien & Greenberg 06

Difference between size distributions of NEOs and main belt likely due to this effect
YORP: Yarkovsky–O'Keefe–Radzievskii–Paddack effect

• Shape and albedo variations affect both spin rate and rotation axis (obliquity) of asteroids. What we talked about previously affected orbit rather than the spin rate and axis.

• Each facet of the asteroid emits light normal to it. Each facet exerts a different torque on the object.
Implications of Yarkovsky and YORP effects

- Orbital element evolution in asteroid belt. Dynamical spreading of asteroid families. Resonance feeding rates and meteorite delivery
- Size distribution differences between NEO and main belt
- Direct measurements with radar: variations in spin, orbital elements
Yakovski effect (continued)

- Asteroids can have varying spin axes. The Yarkovski effect changes the orbits of asteroids, causing them to drift.
- The smaller the asteroid, the greater the Yarkovsky effect. This could explain why the tiniest members of one family of asteroids, known as the Astrids, have the widest range of orbits, (Farinella and Vokrouhlicky note).
- It is possible to estimate the time it takes asteroids to travel from the asteroid belt to the Earth.
- The time that a meteor travelled through space can also be estimated from its cosmic ray exposure time. This is measured by studying the crust of the asteroid.
- Resonances are the major way that asteroids are delivered to earth. However the time it takes for a resonance to deliver an asteroid to earth is shorter than typical cosmic ray exposure times.
- The Yarkovski effect is given as an explanation for the long delivery timescales of asteroids which are required to explain their cosmic ray exposure times.
Gas Drag

Object larger than mean free path of gas particles
The gas acts like a continuum fluid (not like individual particles)
The object experiences a drag force in direction opposite to velocity

\[ F_{\text{drag}} = \frac{1}{2} C_D A \rho v^2 \]

\( C_D \) drag coefficient, unitless

all uncertainties hidden here!

\( A \) cross sectional area
\( v \) velocity
\( \rho \) gas density

\( \rho A \) mass per unit length
\( \rho A v \) mass per unit time swept up
\( \rho A v^2 \) momentum per unit time swept up
Gas drag is important when bodies are passing through a gas cloud or atmosphere.

Particles are also impacted by the solar wind which is very low density.

This drag force is often treated similar to PR drag, and is expected to be most important for the smallest (sub micron sized) particles.

Note: the radiation from the Sun and the solar wind can charge up dust particles. Once they are charged, they can be affected by the magnetic field in the heliosphere.
Some consequences of radiation forces

Comet tails

- Comets usually have two tails, one pointing directly away from the Sun, the other trailing slightly back along the comet’s path.
- The “direct” one, called the ion tail, is made of ionized atoms and molecules, blown almost straight away along the coma-Sun line by the Solar wind.

Some consequences of radiation forces (continued)

- The curved one, called the **dust tail**, consists of dust lifted more gently out of the comet’s coma by radiation pressure. This dust is in orbit around the Sun (with centripetal force balancing gravity *plus radiation force*), so the dust tail is curved in the orbital direction.

Comet 1P/Halley, 8 March 1986, by Bill Liller (IHW/NASA).
Some consequences of radiation forces (continued)

Dust survival

- There is µm-cm size dust in the plane of the Solar system: the zodiacal dust cloud, visible as the **zodiacal light** and the **gegenshein**. It is concentrated in the central Solar system, within Mars’s orbit.

- This dust should be removed by radiation effects in a time much smaller than the age of the Solar system. That it is still there, is an indication that it gets replenished frequently.

- Comet tails and asteroid collisions are the leading producers of dust debris for the Zodiacal cloud.
Summary of forces affecting small bodies, in addition to gravity

• Radiation pressure – μm or less sized particles strongly affected.
• PR drag – on long timescales cm sized particles spiral into the Sun. Particles can be trapped into resonances as they drift.
• Yarkovski effect – affects m – km sized bodies.
• Drag with solar wind – affects particles smaller than a μm.
• Gas drag. Affects bodies if they enter an atmosphere or pass through a gas cloud. More important for smaller bodies. Important near fainter stars with strong winds (AU Mic)
• Electromagnetic forces. Affects charged particles, particularly small ones.